Relaxation through an asymmetric fluctuating potential barrier

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We revise the problem of thermally activated crossing of a fluctuating potential barrier, laying stress on the asymmetry of the barrier. Considering as a working model a paradigmatic triangular dichotomously varying potential landscape we find an untypical dependence of the mean first-passage time on the correlation time τ of fluctuations. Namely, in the range of small τ an additional maximum appears. We propose a qualitative explanation of this feature emphasizing the relevance of dynamics in the vicinity of the barrier top, i.e., recrossing, which generally has not been recognized by this time in this particular context. Moreover, we observe that addition of fast barrier fluctuations of some finite intensity needs not to increase the relaxation rate, as has been indicated many times in extensive studies related to the resonant activation phenomenon. Our findings are confirmed numerically for some other systems.

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I. INTRODUCTION

Thermally activated escape from a potential well has long served as a paradigm of the decay process of metastable states. It is one of the most challenging and long-studied topics in statistical physics and proves a huge variety of applications, e.g., in description of chemical reactions, transport phenomena in biological systems, or nucleation processes [1]. The landmarks in this field were as follow: (i) Formulation of the general law for the duration of characteristic time of the process by Arrhenius in 1889 [2–4],

$$T_0 \sim \mathcal{A} \exp\left(\frac{E}{k_B \mathcal{T}}\right), \qquad (1)$$

for $k_B \mathcal{T} \ll E$ with E, $\mathcal{T}$, and k_B being the activation energy (potential barrier height), temperature, and Boltzmann constant, respectively. (ii) Description of the problem by means of Brownian motion due to thermal noise in a field of force and presentation of its solution for strong and intermediate-to-weak friction by Kramers in 1940 [3]. Kramers' idea consists in an observation that for high enough barrier the quasiequilibrium in the potential well is attained much before the Brownian particle can leave it. Thus, the activation time is related mostly to the duration time of staying in the region of bottom of the potential well, while the duration t_s of the very escape event (climbing to the barrier top) is comparatively shorter. (iii) Construction of the turnover theory by Pollak, Grabert, and Hänggi in 1989 [5], which bridges the gap between the strong and weak friction limits. The most essential part of this theory consists in the detailed analysis of the recrossing phenomenon of the region of the barrier top, which the Brownian particle undergoes.

A new impact in the research of the activation problems has arisen some 20 years ago when it has been realized that an additional time-dependent perturbation of the Brownian particle can lead to some unexpected astonishing results. The mostly known examples are stochastic resonance [6,7], transport in ratchetlike systems [8,9], noise-enhanced stability of metastable states [10], or resonant activation (RA) [11]. The latter effect concerns the activation process in stochastically varying potential landscapes that happens, e.g., in systems with many degrees of freedom such as protein molecules [12] and glasses [13] or in systems with externally controlled parameters subject to fluctuations such as colloids in optical potential [14] or semiconductor lasers [15]. Theoretical studies of the problem have been undertaken by a number of authors [13,16–20] and have eventually led Doering and Gadoua [11] to the discovery of the phenomenon of RA, i.e., the appearance of a minimum of the mean first-passage time (MFPT) as a function of the correlation time τ of the potential perturbation. Further studies [21–27] have proven the universality of that phenomenon. It can appear for different statistics of potential perturbations [26,28,29], for periodic barrier modulation [30,31], or for different spatial forms of potential and perturbation [24,32]. RA can be also identified in relaxation of metastable states in nonpotential systems [33,34] or in quantum dynamics of open systems [35,36]. Recently it has been shown both theoretically [37] and experimentally [38] that the RA phenomenon may also appear due to correlated fluctuations of the temperature of the surrounding.

For the purposes of this paper we shortly summarize main results of these investigations using the notion of the MFPT over the potential barrier (cf. Fig. 1). For any Markovian exponentially correlated barrier noise, when its variance σ^2 is kept constant (CVS), infinitely fast fluctuations ($\tau \to 0$) have vanishing intensity [39],

$$Q = \sigma^2 \tau. \qquad (2)$$

Thus, the value of the MFPT is exactly the same as for the static barrier T_0, which for weak enough thermal noise is given by Eq. (1). When τ grows from 0 it is argued that barrier fluctuations can be treated mostly as an apparent local increase in the thermal noise intensity, which leads to the decrease in MFPT. In the opposite limit $\tau \to \infty$, MFPT results as the ensemble average of the MFPTs over static barriers distributed according to the statistics of potential fluctua-

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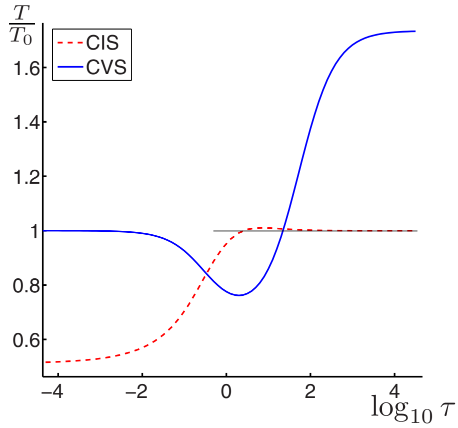

FIG. 1. (Color online) MFPT over a fluctuating potential barrier (normalized to MFPT over the static barrier) vs the correlation time: typical shapes for constant variance scaling (CVS) and constant intensity scaling (CIS). Thin horizontal line corresponds to the level of T_0.

tions. The obtained value T_∞ is larger than T_0 due to convexity of the exponential function [Eq. (1)]. As a consequence of these features a minimum of MFPT as a function of τ has to exist (Fig. 1). It arises for the value of τ that is approximately equal to the duration of time t_s, in which the escaping particle climbs up the slope of the barrier [40,41]. For this value of τ the Brownian particle escapes with the prevailing probability when the barrier is as low as possible for given statistics of the potential fluctuations, which means lower than the height of the unperturbed barrier [41]. This yields a resonantlike character of the phenomenon.

For CIS the variance $\sigma^2=Q/\tau$ vanishes in the limit $\tau\rightarrow\infty$ and hence $T_\infty=T_0$. When $\tau\rightarrow 0$ the noise becomes uncorrelated (white), which enlarges the effective diffusion and hence MFPT becomes lower than T_0. This limit is singular with respect to the noise variance, which becomes infinitely large. A nonzero value of τ (finite memory of the noise) yields smaller (finite) amplitude, which results in an increase in MFPT in the range from small to moderately large correlation times. Thus, a minimum characteristic for the RA does not appear. However, resonantlike effect of potential fluctuations on the activation process is revealed in the statistics of barrier heights which are surmounted during the escape events [41]. Let us mention finally that for rather large τ a small maximum of the MFPT can appear [25,26]. It has been called *inhibition of activation* [25], but it is rather of geometrical than resonant nature [41].

In the following we are going to reconsider these known arguments and show that they are not unconditionally applicable while dealing with passage between two potential wells through an asymmetric barrier. After defining the model, we first analyze the case of CVS, which seems to be better recognized and simpler in an analysis. Next, we turn to the CIS case, which appears to be more intriguing and difficult to handle. Then we discuss the influence of barrier fluctuations on the stability of the system and robustness of our observations against some extensions of the basic model and, finally, we sum up our results and present some general remarks.

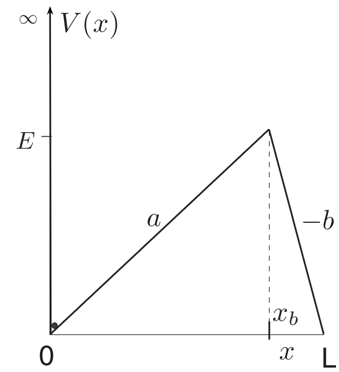

FIG. 2. Piecewise linear potential $V(x)$.

II. MODEL

We consider an overdamped Brownian particle initially placed at the point $x=0$, which is a metastable state of the piecewise linear potential displayed in Fig. 2. The dynamics of the particle (after scaling out the viscosity) is governed by the following Langevin equation:

$$\dot{x} = -V'(x) + \sqrt{2q}\xi(t). \tag{3}$$

Thermal fluctuations are represented by Gaussian white noise $\xi(t)$ with the correlation $\langle\xi(t)\xi(s)\rangle=\delta(t-s)$ and the intensity $q=k_B\mathcal{T}$. A reflecting boundary is assumed at $x=0$ and the relaxation process is completed when the particle surmounts the potential barrier and reaches the point $x=L$ behind it (absorbing boundary). The left- and right-hand side slopes of the barrier are $a=E/x_b$ and $b=E/(L-x_b)$, respectively. If $x_b=L/2$ both they are the same and the barrier is symmetric—the case which has usually been studied previously [11].

The Fokker-Planck equation associated with Eq. (3) reads $\partial P(x,t)/\partial t=L(x)P(x,t)$, where $L(x)=\partial/\partial x V'(x)+q\partial^2/\partial x^2$ and $P(x,t)$ is the probability density to find the particle at the point x at the time moment t. The MFPT T_0 from $x=0$ to $x=L$ can be calculated in the standard way [42] from the solution of equation $L^\dagger T(x)=-1$ with the boundary conditions $T'(0)=0$ and $T(L)=0$. We obtain

$$T_0 = T_{0\rightarrow L} = \frac{a+b}{b}\left\{\frac{q}{a^2}[\exp(E/q)-1]-\frac{x_b}{a}\right\} + \frac{a+b}{a}\left\{\frac{q}{b^2}[\exp(-E/q)-1]+\frac{L-x_b}{b}\right\}. \tag{4}$$

Now we pass to the situation when the barrier fluctuates. We assume the following form of the Langevin equation:

$$\dot{x} = -V'(x)[1+z(t)] + \sqrt{2q}\xi(t), \tag{5}$$

where $z(t)$ is a symmetric Markovian dichotomous noise (DN) characterized by the rate $\gamma=1/(2\tau)$ and, in accordance with Eq. (2), either by the variance σ^2 or by the intensity Q. Its correlation function reads $\langle z(t)z(s)\rangle=\sigma^2\exp(-|t-s|/\tau)$.

As mentioned before, in the limit of zero correlation time for CVS the noise $z(t)$ vanishes and the MFPT equals T_0, whereas for CIS one has to consider Langevin evolution with an effective x-dependent diffusion,

$$\dot{x} = -V'(x) + \sqrt{2\{[q + QV'^2(x)]\}}\zeta(t), \tag{6}$$

where $\zeta(t)$ is the effective Gaussian white noise with the correlation $\langle\zeta(t)\zeta(s)\rangle = \delta(t-s)$. This equation should be interpreted in the Stratonovich sense [42]. The expression for the MFPT of this problem reads

$$T_w = \frac{q+Qa^2}{a^2}\left[\exp\left(\frac{E}{q+Qa^2}\right) - 1\right] - \frac{x_b}{a} + \frac{q+Qb^2}{b^2}\left[\exp\left(\frac{-E}{q+Qb^2}\right) - 1\right] + \frac{L-x_b}{b} + \sqrt{\frac{q+Qa^2}{a^2}}\sqrt{\frac{q+Qb^2}{b^2}}\left[1 - \exp\left(\frac{E}{q+Qa^2}\right)\right] \times \left[\exp\left(\frac{-E}{q+Qb^2}\right) - 1\right]. \tag{7}$$

If $\tau > 0$ the MFPT is given as $T = [T_+(0) + T_-(0)]/2$, where the conditional MFPTs, $T_\pm(x)$, correspond to the initial value $z(0) = \pm\sigma$ of the barrier perturbation and they fulfill the following set of equations [11]:

$$-(1+\sigma)V'(x)T'_+ + qT''_+ - \gamma T_+ + \gamma T_- = -1,$$

$$-(1-\sigma)V'(x)T'_- + qT''_- + \gamma T_+ - \gamma T_- = -1, \tag{8}$$

with the boundary conditions

$$T_\pm(L) = 0, \quad T'_\pm(0) = 0 \tag{9}$$

and the continuity conditions for $T_\pm(x)$ and $T'_\pm(x)$ at x_b. In principle, for the triangular potential given in Fig. 2 this set of equations is solvable analytically. However, the exact results are much too complicated to be presented, not to mention to draw any useful conclusions from them [11], so in what follows we present numerical solutions of Eq. (8) only.

III. CONSTANT VARIANCE SCALING

For the CVS case the dependence of the MFPT on τ for different asymmetries is displayed in Fig. 3. As expected, each curve exhibits an RA minimum, which is shifted to the right for potentials with longer left slope. This is mostly because the duration of the instanton time t_s is longer for longer slopes (cf. Sec. I and [40]). More careful inspection of the region of small τ makes it clear that for ascending values of x_b a small maximum to the left of the RA minimum can also appear (see the inset of Fig. 3). To check this effect more carefully we solve system (8) perturbatively for small values of the correlation time. The method consists in expanding the eigenvalues of the left-hand side of set (8) rather than the very set of Eq. (8) in powers of τ and it has already been used to investigate some related problems [37]. The result reads

$$T \simeq T_0 + \tau T_1 + \cdots, \tag{10}$$

with T_0 given by Eq. (4) and

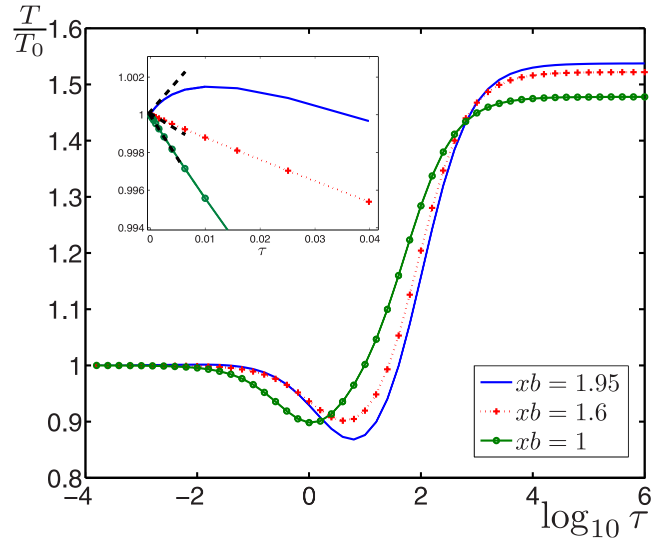

FIG. 3. (Color online) MFPTs for different barrier asymmetries vs τ for CVS. We set $L=2$, $E=0.2$, $q=0.08$, and $\sigma^2=0.5$. Inset: range of small τ. Dashed lines represent approximations (10) and (11).

$$T_1 = \sigma^2\left\{[\exp(E/q) - 1]\frac{L}{x_b}\left[\frac{L}{2(L-x_b)} - \frac{E}{q}\right] + [\exp(-E/q) - 1]\frac{L}{L-x_b}\left[\frac{L}{2x_b} + \frac{E}{q}\right]\right\}. \tag{11}$$

For $E > 0$ the second term in curly brackets of Eq. (11) is always negative. However, for high enough barrier or low enough noise and for x_b close enough to the end $x=L$, the first term can be positive and can dominate the second one, so T_1 can be positive. Consequently, one may observe that MFPT initially grows with τ up to a small maximum, as is presented in Fig. 3. Excluding the multiplicative factor σ^2, the first-order term T_1 depends *only* on the ratio $\theta = E/q$ and on the relative position of the barrier $s = x_b/L$, which can be considered as the parameter of the barrier asymmetry. One can find that T_1 changes its sign for

$$s = \frac{2\theta - (1-e^{-\theta})}{2\theta(1-e^{-\theta})}. \tag{12}$$

From this relation one can infer the necessary conditions for the existence of the maximum. Namely, the barrier has to be high enough, $\exp(\theta) \geq 1 + 2\theta$ (i.e., $\theta \geq 1.256$), as well as asymmetric enough, $1 > s \geq s_{\min} = 0.886$ with $s_{\min}$ given by Eq. (12) for $\theta_1 = 2.983$ being the solution of the equation $\cosh(\theta_1) = 1 + \theta_1^2$.

What is the origin of the emergence of that small maximum? It is usually argued that an addition of a multiplicative noise of extremely small correlation time and finite variance to the system results mainly in a local increase in effective temperature, which accelerates the escape process [39]. Indeed, the MFPT for a static triangle barrier with a slope a over the interval $[0, x_b]$ with the reflecting and absorbing boundary conditions at $x=0$ and $x=x_b$, respectively, reads

$$T_0^s = \frac{q}{a^2}[\exp(ax_b/q) - 1] - \frac{x_b}{a}. \tag{13}$$

The corresponding first-order correction can be obtained by substituting $q+\tau\sigma^2 a^2$ in place of q in formula (13) and subsequent expansion in powers of τ. This gives

$$T_1^s = \sigma^2\left[\exp(ax_b/q)\left(1 - \frac{ax_b}{q}\right) - 1\right], \tag{14}$$

which is always negative. The same holds true also for the MFPT through a descending slope. So, at least when the barrier is not to small, a particle, which succeeds in passing a small distance behind the barrier top at x_b, will also complete the escape process sliding down faster. Let us notice that for CVS scaling first-order correction (11) can be obtained in a similar way as Eq. (14) substituting Eq. (2) to Eq. (7) and expanding in τ. Because small but nonzero correlation time always accelerates the evolution on both slopes of the triangle barrier, but under some conditions T_1 becomes positive, we conclude that the observed slowdown is caused by the transition between both slopes through the region of the barrier top.

In general, it is more likely for the particle located at the top to move along the steeper slope, i.e., in our case to the right. On the other hand, since we are considering multiplicative barrier fluctuations, so the increase in the effective temperature with τ is more significant on the steeper slope. Hence, the probability that the particle, which has appeared in the vicinity of the barrier top, encounters a fluctuation large enough to push it back through the top rises more significantly for larger slope of potential. This is why the probability to go along the steeper slope decreases. When the left slope is more gentle, this results in an increase in probability of recrossing back to the left.

The conclusions from this section, which foreshadow some more significant anomalies in the following parts, are as follow. For small τ we must consider two important factors: enhancement of diffusion and enhancement of recrossing probability. In some range they coexist and compete. For large asymmetry parameter s the second effect prevails and then the maximum occurs. However, when τ is larger—say, of the order of magnitude of t_s for the first slope—the correlation of the DN becomes important and as usual we observe a minimum corresponding to RA.

Since for not to large variances σ^2 the maximum is rather small, the shape of $T(\tau)$ does not differ dramatically from the symmetric or bottom-top case. We note that in contrary to what was tacitly assumed in the literature [24,25,43], for an asymmetric barrier one cannot simply announce the absence of the RA minimum checking the sign of the leading-order term of MFPT for small τ. Moreover, extremely fast (and small) barrier fluctuations need not to result in an enhancement of the decay rate of a metastable state.

IV. CONSTANT INTENSITY SCALING

For the commonly considered bottom-top evolution (or a symmetric barrier case), when the noise intensity Q does not depend on the correlation time, no minimum is observed in the τ dependence of the MFPT. A question arises if any anomalous feature appears when the barrier is asymmetric. Figure 4 answers this question.

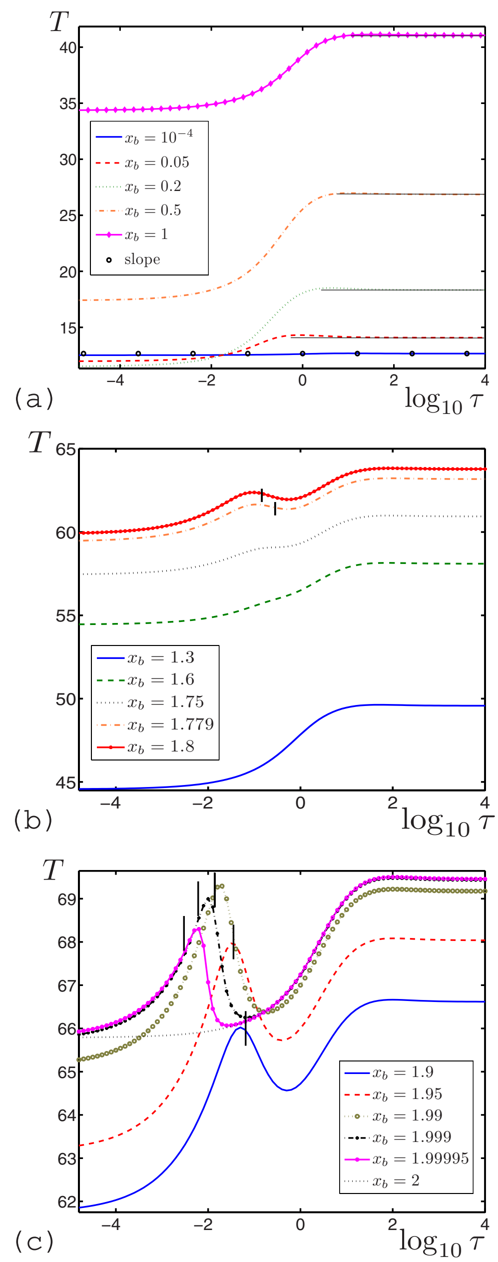

FIG. 4. (Color online) MFPT vs τ for CIS for long (upper panel), short (middle), and very short (lower panel) second slopes. Q=0.2 and the other parameters as in Fig. 3. Here and in the following pictures short vertical bars denote estimation of the location of the maximum according to Eq. (17). Thin horizontal lines in the upper panel denote the corresponding values T_0 of MFPTs over static barriers.

When the left slope is shorter or the potential is symmetric, nothing untypical is observed [Fig. 4(a)]. For x_b close to x_0=0 the weak maximum interpreted as an inhibition of activation [25] is a little bit more pronounced and as $x_b \to 0$ we eventually recover the MFPT along a sole slope of length L and height E.

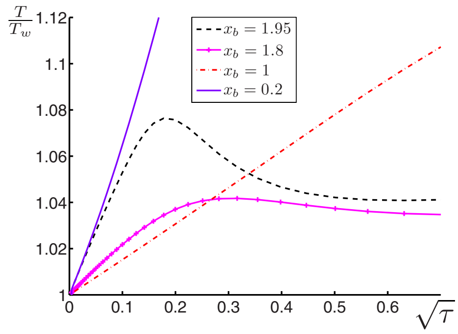

FIG. 5. (Color online) Relative MFPTs for different asymmetries and CIS: small τ regime. Parameters as in Fig. 4.

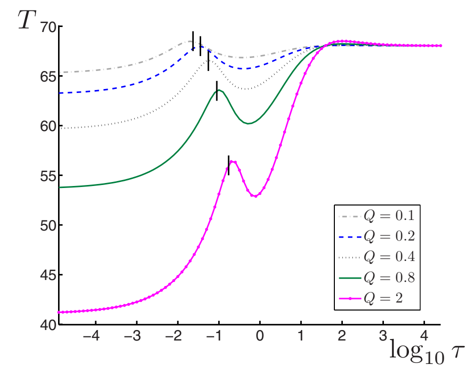

FIG. 6. (Color online) MFPT for CIS and different intensities Q. The barrier is located at x_b=1.95; the other parameters are as in Fig. 3.

However, for asymmetries corresponding to a long first slope [Fig. 4(b)], the curve $T(\tau)$ deforms itself as x_b increases, which gives rise to the appearance of a clear maximum and a subsequent minimum, which are finally followed by the maximum of inhibition of activation. A further increase in the asymmetry makes the maximum more significant [Fig. 4(c)].

It would seem that the appearance of this maximum results simply due to the high asymmetry of the barrier. In fact, the nature of this effect is much more subtle. Namely, as we can see in Fig. 4(c) for very large asymmetries the peak of the maximum decreases and in the limit $x_b \to L$ one eventually gets the curve of MFPT for climbing a single-slope barrier of the height E and width L, the form of which has a well known shape without any other extremes beside the one of inhibition of activation. Therefore, for a range of small flipping times τ there exists a critical form of the barrier asymmetry, for which the activation process is slowed down by stochastic barrier perturbation $z(t)$ most strongly.

Exploiting the same method as in Sec. III one finds the following expansion of $T(\tau)$ for small values of correlation time:

$$T \cong T_w + \sqrt{\tau} T_{1/2} + \tau T_1 + \cdots, \qquad (15)$$

with T_w given by Eq. (7). Looking at Fig. 5 one might hope to find the position of the maximum from a parabolic approximation of $T(\sqrt{\tau})$, but unfortunately, the expressions for $T_{1/2}$ and T_1 are very long and completely cumbersome. We only notice that the condition on asymmetry to be imposed for the presence of a maximum is here less restrictive than in the case of CVS. For the parameters as in Fig. 4 the critical point inferred from Eq. (12) would be x_b=1.779. This corresponds to the dashed-dotted line in the middle panel of Fig. 4, for which one can see an already well pronounced maximum.

As it has been mentioned, the new maximum of the MFPT appears for the range of rather small values of the correlation time. Its actual position $\tau_{\max}$ depends on the value of the intensity of the barrier noise, moving slightly to larger τ as Q increases (Fig. 6). Nevertheless always $\tau_{\max}$ remains so small that due to relation (2) the amplitude of the noise is much greater than the slope of the unperturbed barrier. This means that, literally speaking, the region of the barrier maximum is being deformed so strongly that temporarily the barrier changes into the well. We thus proceed to justify the observed slowdown of the activation for CIS considering the behavior of stochastic trajectories in such instantaneous well.

The hypothesis, that it is stochastic dynamics of the vicinity of x_b which plays a crucial role in the appearance of the maximum, is clearly supported by Fig. 7, in which MFPTs from x_0=0 to $x_b(T_{0\to x_b})$ and to $L(T_{0\to L})$ are displayed. As expected, their values are not very different from each other

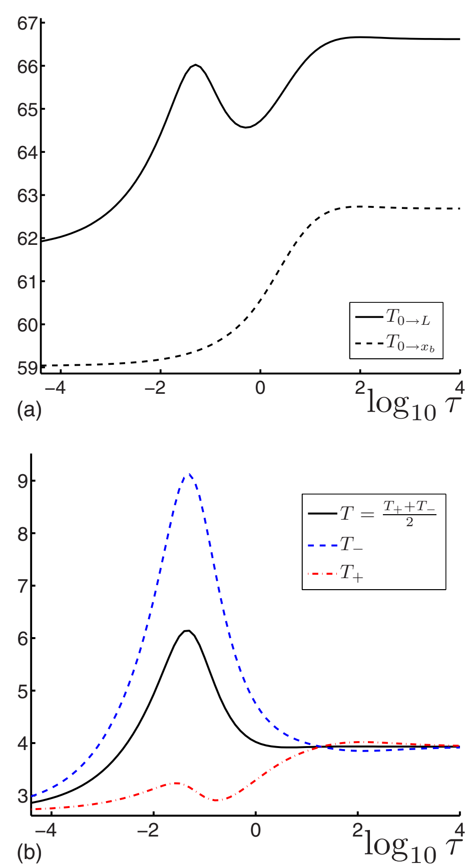

FIG. 7. (Color online) Upper panel: MFPTs from 0 to x_b and from 0 to L for x_b=1.9, Q=0.2 and the remaining parameters as in Fig. 3. Lower panel: conditional and full MFPTs from x_b to L for the same system. In both cases reflecting boundary is situated in point 0.

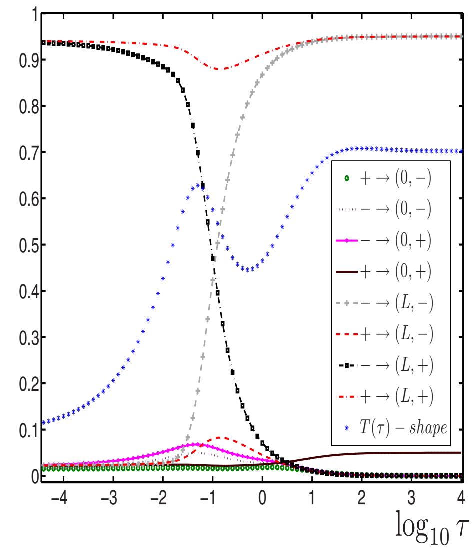

FIG. 8. (Color online) Splitting probabilities for a Brownian particle starting at x_b in a fluctuating potential profile. "$+\to(0,-)$" corresponds to the probability that particle starting from x_b when the barrier is in high configuration will leave the segment $(0,L)$ through the left boundary in the low configuration. Other probabilities are specified in a similar manner. Parameters as in Fig. 7. The shape of the curve $T_{0\to L}$ is displayed for the reader's convenience only.

since x_b is located close to $x=L$. For large τ, for which the potential is almost not perturbed, they differ mainly by a multiplicative factor of the order of unity which reflects the typical fluctuations appearing in the vicinity of the barrier top which led to multiple recrossing of this region. Comparison with the lower panel of Fig. 7 reveals evidently that the unexpected growth of $T_{0\to L}$ in the range of small τ is related to some delay of the escape process of particles reaching the top of the static barrier at x_b. More precisely, in some range of flipping frequencies there is a mechanism of trapping those particles which reach x_b when the barrier is lowered. This process is significantly elucidated by Fig. 8, in which splitting probabilities (Chap. 5.2.8 in [42]) for particle initially located at x_b are displayed. When switching is extremely fast ($\tau\to 0$), the initial configuration is inessential. Since + configuration corresponds to a very high peak and − to a very deep well, both very asymmetric, the escape is finalized with likelihood close to certainty through sliding down right in the + configuration. Actually, for the initial state being +, this statement remains true in all range of values of τ: the slope is so steep that once a small step is done to the right from x_b, then the particle escapes extremely quickly. But we have to keep in mind that with an increase in τ the particle reaches x_b from the left more likely in the − configuration (maximum of that likelihood corresponds to RA), so we should consider more carefully just this configuration as an initial state.

When τ grows from zero, a particle which reaches x_b is trapped in a very deep instantaneous well in average for a period of duration of $\gamma^{-1}=2\tau$, with practically no chance to escape. It waits there until the barrier flips, so that it might slide down to the right. But at the same time, as τ grows, the thermalization process in the well becomes more and more significant. Because the valley has a highly asymmetrical shape, so the majority of the probability distribution is located to the left from x_b not only in its closest vicinity. Thus, after the barrier switches the particles are able to slide down to the left with an increased probability. This is confirmed by Fig. 8, which displays that the likelihood of escape to the left from the initial configuration − grows with τ for both final configurations (dotted line and solid line with dots). Still, even when these splitting probabilities reach their maximal values, it is by many times more probable to escape to the right through the + configuration (dashed-dotted line). One must, however, remember that this latter process lasts negligibly shortly. That is why even a small amount of particles, which have to slide down to $x_0=0$ and climb up again in order to get another chance to complete the relaxation process, can increase the value of the MFPT substantially.

Following this scenario of the diffusion process we can make a rough estimation of the value of the MFPT. First, for the used data one can estimate from Figs. 7 and 8 that a single sliding-down event lasts about 20 time units (t.u.), while for climbing up one needs about 60 t.u. Hence, neglecting for simplicity multiple reversions, the overall return process back to x_b needs about 80 t.u. Next, the total probability of reaching the left boundary prior to the right one is a sum of two splitting probabilities defined for both initial potential profiles. They are marked by dotted line and solid line with dots on Fig. 8, and their maximal values are approximately 0.05 and 0.07, respectively. Thus, we obtain $80\times 0.12=9.6$ t.u. as our estimation of the value of the MFPT at its maximum, which coincides pretty well with the maximal value of mean escape time from x_b with the initial configuration − depicted in the lower panel of Fig. 7. Similar estimations can also be done in other regimes of the correlation times. This maximum evidently coincides (just as the maximum of the full MFPT from $x_0=0$ does) with the maxima of the splitting probabilities.

With increasing τ the relaxation process within a temporary well becomes more effective, and one might have expected some saturation when a quasiequilibrium distribution is approached. This is not the case—for the correlation time longer than some critical value τ_c the MFPT starts to decrease (Fig. 7). This is because under CIS scaling the amplitude of the fluctuations gets smaller, so that for $\tau>\tau_c$ the instantaneous valley becomes shallow enough for the Brownian particle to escape diffusively to the right within the time interval 2τ. In Fig. 8 we see a sudden drop of the probability to escape to the right in the + state (and also a decline of the remaining splitting probabilities) in favor of the − configuration. To a minor degree this concerns also the probabilities for the initial configuration being +, with smaller relevance for the whole slowdown process. This feature results in breakdown of the trapping phenomenon and the MFPT begins to decrease.

Can we make any quantitative predictions about the location of the maximum? According to what is stated above, the MFPT should start to decrease when $2\tau=t_{esc}(\tau)$, where t_{esc} is the mean time to escape from the instantaneous well when the initial position is x_b. It depends on τ since the well's

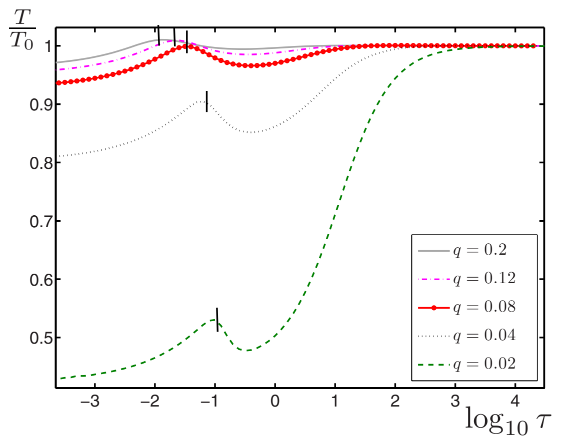

FIG. 9. (Color online) Dependence of MFPT on τ for various thermal noise intensities q. We set $Q=0.2$, $x_b=1.95$; other parameters as in Fig. 3.

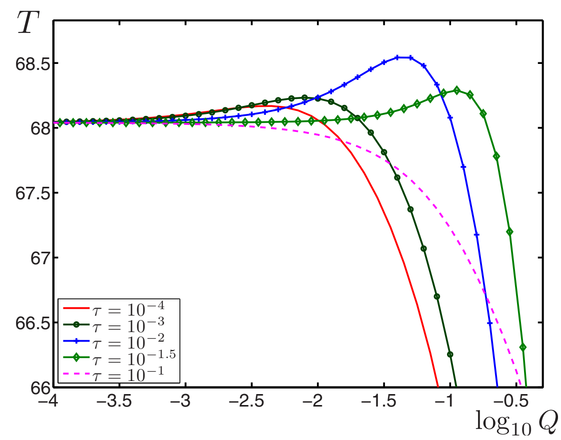

FIG. 10. (Color online) MFPT vs dichotomic noise intensity Q for different correlation times. The barrier is located at $x_b=1.95$; the other parameters are as in Fig. 3.

depth is τ dependent. Taking for t_{esc} the MFPT for a static valley with reflecting boundary at 0 would be highly misleading, however. This is because the left slope is very long compared to the right one and relaxation processes are very slow there. As a result, a major contribution to such MFPT would come from this fraction of the ensemble of trajectories which wander very long on the left slope.

Let us consider formula (13) for the MFPT on a linear slope of potential once again. Using $E=ax_b$ one may rewrite it as

$$T_0^s = \left[\frac{q}{E^2}(e^{E/q}-1) - \frac{1}{E}\right] x_b^2. \qquad (16)$$

This means that the MFPT to reach the top of a triangle barrier of the height E depends on square of the distance to be passed. If one inserts a reflecting boundary at the bottom $x=x_b$ of the instantaneous valley the mean time to reach the point $x=0$ is much longer (two orders of magnitude for our data) than the time to arrive at $x=L$. Thus, within the time interval needed to escape to the right those particles which are to the left from x_b are still concentrated in the vicinity of the bottom of the valley. This allows us to estimate t_{esc} as the MFPT from x_b to L with the reflection boundary at x_b. From Eq. (16) one gets

$$2\tau = \frac{q(L-x_b)^2}{E^2(\sqrt{Q/\tau}-1)^2}\{\exp[E(\sqrt{Q/\tau}-1)/q]-1\} - \frac{(L-x_b)^2}{E(\sqrt{Q/\tau}-1)}. \qquad (17)$$

Its solution (for τ) can be easily found numerically and is marked in Figs. 4, 6, and 9 by vertical bars. We can see that we receive in this way a useful tip where the maximum can be found. It persists in giving reasonable estimations against variations in different parameters (cf. Figs. 6 and 9) and fails only for extremely small intensities Q and high asymmetries.

As τ grows further, the amplitude of fluctuations is not large enough to create a temporary well form of the potential anymore and the escape process is brought to the end (from x_b to L) mostly through sliding down in the initial configuration (cf. Fig. 8). Thus, the MFPT from x_b to L reaches its asymptotic value (cf. Fig. 7) and the evolution in this part of potential does not play longer any important role. But simultaneously, the MFPT from $x_0=0$ to x_b grows very fast with τ since the particle does not benefit further from sliding down in the lower configuration. Hence, for longer τ the curve $T(\tau)$ has the typical shape described in Sec. I.

V. ENHANCEMENT OF STABILITY OF METASTABLE STATE

An astonishing aspect of the feature found in Sec. IV is the fact that for high, but not to much, asymmetries an additional driving of the system with a fast colored noise practically does not change the decay rate (cf. plots for $x_b = 1.95, 1.99$ in the lowest panel of Fig. 4 or for $Q=0.1, 0.2$ on Fig. 6). To be more precise, we have shown that for small but finite ($Q>0$), fast enough (τ is smaller than the relaxation time inside the unperturbed well) correlated ($\tau>0$) noise, provided some combinations of the remaining parameters, the MFPT approximately equals that of the particle in the unperturbed system T_0. Moreover, even a small delay of the relaxation can be observed (Figs. 3 and 6). This phenomenon is illustrated in Fig. 10, in which the dependence of the MFPT on the multiplicative noise intensity is presented. An increase in the noise level Q initially makes the metastable state more sustained, while eventually, for large enough Q, the relaxation process is abruptly accelerated by the colored noise. As it is seen from the plot there exist optimal values of τ and Q for which the delay of the decay process is largest.

Looking at Fig. 10 we notice the hallmarks of the phenomenon of noise induced stability, reported for a dozen of years in different contexts (cf. first of all [10,44] and further references therein). However, apart from many details, also some fundamental differences are clearly visible. Whereas in studies of Valenti *et al.* [45] it has been inessential for the appearance of the effect whether the perturbing noise is white or colored, here apparently a rather intricate interplay of finite correlation time with system parameters is, in general, necessary for the stabilization to occur. Next, in the works of Valenti *et al.* it has been the thermal (white) additive noise which has slowed down the decay process when the system has been subjected to the dichotomic noise. In our study the multiplicative dichotomous noise does play the crucial role, while the white Gaussian noise is merely an assisting factor.

VI. DISCUSSION AND SUMMARY

In so far, we have demonstrated the existence of unexpected maxima only for a simple model of dichotomously perturbed triangle barrier [11,21]. One can unavoidably ask how robust these effects are against various modifications and what is their nature.

We have studied numerically some modifications or generalizations of the present model: (i) a shift of the absorbing boundary up or down with respect to the $E=0$ level; (ii) a smooth-form potential instead of the piecewise linear; (iii) some other types of scaling of the colored noise intensity giving a characteristic intermediate between CVS and CIS [25]; and (iv) other than dichotomous perturbations of the barrier. In each case the general features of the described effect have been confirmed. We mention some details of the last case. Namely, we have performed calculations for pure-jump process with binomial distribution of variance σ^2 composed of k independent Markovian dichotomic processes, which approaches in the limit $k \to \infty$ the Ornstein-Uhlenbeck noise [46]. Increasing k we observe fast convergence of the calculated MFPT to its limit form (Fig. 11), which allows us to conclude that a maximum of MFPT corresponding to slowdown of activation is present also for continuous Gaussian fluctuations of the potential.

By now we have neither suggested nor excluded resonant character of the slowdown reported in this paper. In fact, as clarified in Sec. IV, there is some adjustment of time scales here. It involves the correlation time τ and internal dynamics of an instantaneous "virtual" potential well. But since this temporal confinement is noise induced, we do not deal with any simple resonant coupling of the unperturbed system to the external noise.

To sum up, the main finding of this paper states that providing high enough asymmetry of the shape of the potential barrier a maximum of mean first-passage time in the range of small correlation times of the colored noise can appear, notwithstanding the intensity scaling of the barrier perturbation. For constant variance scaling we have presented some analytical calculations and formulated some conditions for the appearance of this maximum. For constant intensity of the fluctuations we have explained the effect by means of splitting probabilities and we have proposed a heuristic rule for finding the location of the maximum of MFPT. The considered generalizations of the basic setup allow us to put forward the hypothesis that we have identified a generic property of relaxation phenomena in spatially asymmetric systems subjected to colored multiplicative noise sources.

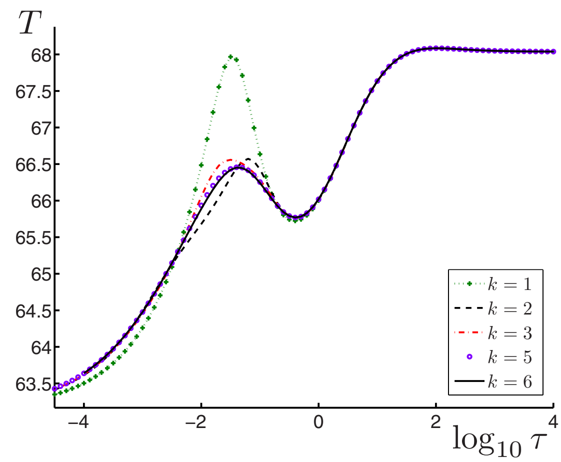

FIG. 11. (Color online) MFPT over an asymmetric piecewise linear barrier vs correlation time τ perturbed by pre-Gaussian noises composed of $k=1,2,3,5,6$ dichotomic processes. $Q=0.2$, $x_b=1.95$; the other parameters as in Fig. 3.

The detailed consideration of recrossing events has been the key to understand the found effects. It has appeared that if the barrier itself undergoes stochastic fluctuations, the random dynamics in the very vicinity of the barrier top may result in a rather unexpected behavior. Namely, we have identified an extra maximum in the dependence of the MFPT on the correlation time of the perturbation. We have also demonstrated that fast barrier fluctuations of small intensity not necessarily reduce the stability of metastable states. On the contrary, they can prolong the lifetime of metastability.

Let us note finally that quite recently Fiasconaro [47] has studied the role of asymmetry in fluctuating barrier crossing. However, he has considered only slightly asymmetric barriers, hence he has not mentioned the possibility of the occurrence of a maximum of the MFPT for small τ.

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